The Geometric Moving Average Control Chart: A Full-Purpose Process-Control Tool

ASQ-Baltimore Section Meeting

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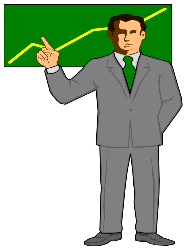
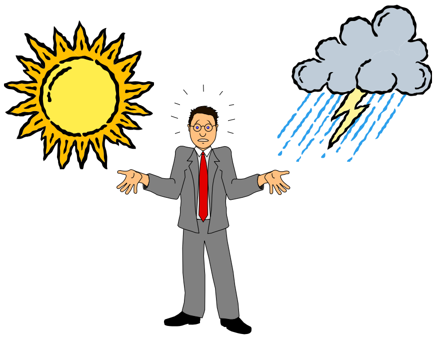
Focus: SPC Forecasting Control Chart





Outline

- Introduction
- Classical vs. Modern views
- Comparison between charts
- Constructing GMA charts
- Example
- Conclusions

SPC Popular Due to

- Successful in Pre-1940s Technology
- Japanese Advances in Quality
- Regain Competitve Edge in 1980s

In Catching Up We Found

- Automated Systems Expanded
- “Older Tools” Inadequate
- Modern Tools Not Fully Used
- Modern Methods Adopted/Adapted

Classical SPC "Passive"

- Describes What Happened
- Curative in Approach
- Monitors Stability

Modern SPC “Active”

- Analyzes Causes & Effect
- Predicts Future
- Gives Dynamic Feedback/Feedfront
  Basis for Action

Standard Control Chart Schemes

- Shewhart
- Individuals, Moving Range, & Moving Average
- CUSUM
- GMA (EWMA)

Shewhart Control Charts

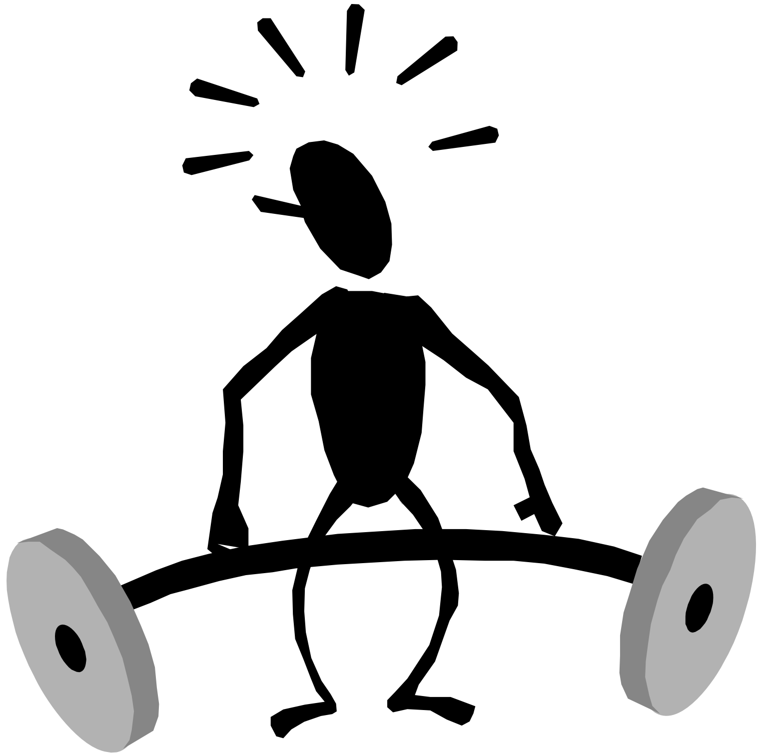
- Developed by Dr. W.A. Shewhart (1924)
- Monitors Process Stability with Control Limits
- Points Outside Limits Signal Instability
- Places “Weight” on Latest Observation

Shewhart Charts Pros

- Easy to Construct
- Detects Large Process Shifts
- Helps find process change caused so that countermeasures against adverse effects may be implemented

Shewhart Chart Cons

- Ignores History
- Hard to Detect Small Process shifts
- Hard to Predict In/Out-of-Control Processes
- Really is not Process CONTROL, but is used for RESEARCH Control



Individuals, Moving Range & Moving Average Charts

- Special-Case Shewhart Charts
- Single Observations
- Control signal Like Shewhart

Individual (I) Charts

- Few Units Made
- Automated Testing on Each Unit
- Expensive Measurements
- Slow Forming Data
- Periodic (Weekly, Monthly) Adminstrative Data

Moving Range (MR) Charts

- Correspond to Shewhart Range charts
- Used with Individuals
- Plot Absolute Differences of Successive Pairs (Lags)

Moving Average (MA) Charts

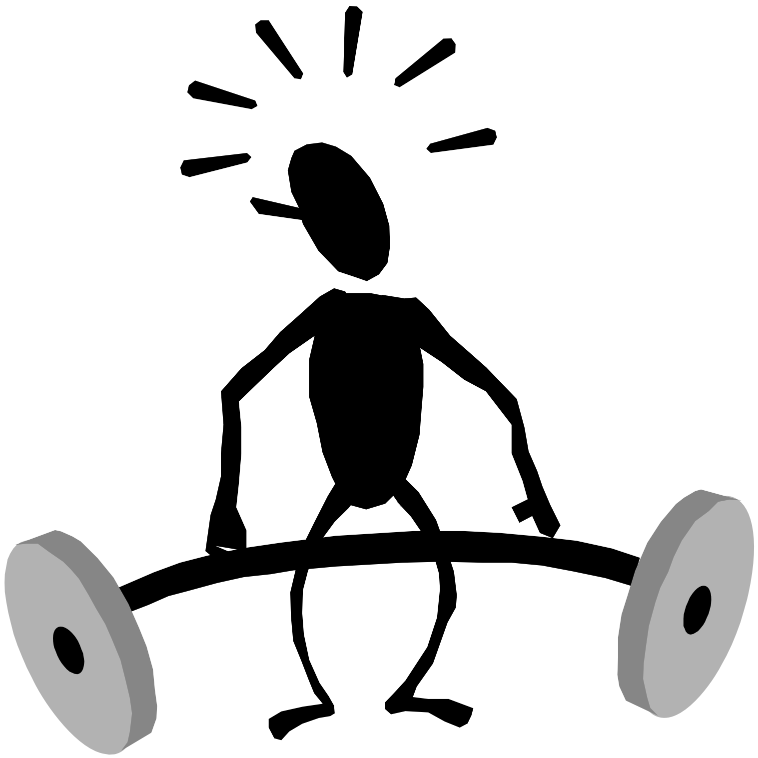
- Running Average of (=4,5) Observations
- New MA Drops “Oldest” Value
- Weight Places like Shewhart (n=2 for MR; n=4,5 in MA; 0 Otherwise)

I/MA/MR Pros

- Detects Small Process Shifts
- Uses Limited History

I/MA/MR Cons

- Auto(Serial) correlated Data Mislead Process Stability
- Show Nonrandom Patterns with Random Data

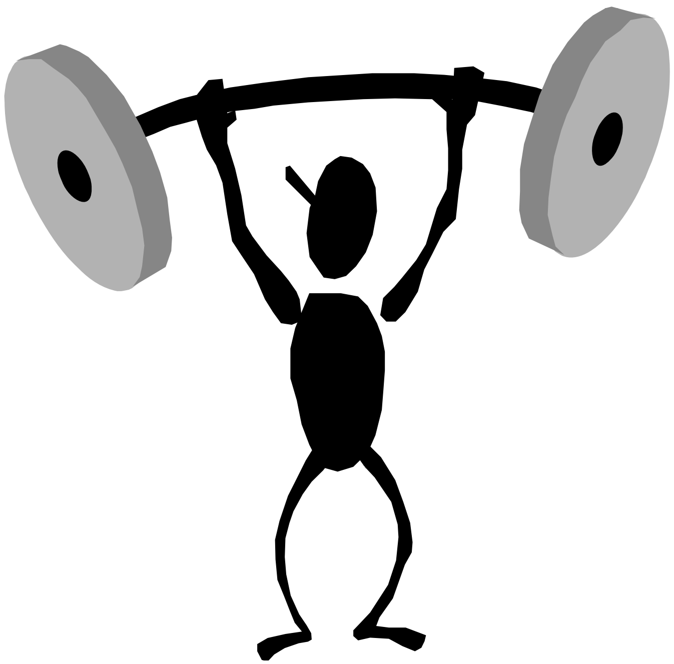


Cusum Control Charts

- Introduced by E.S. Page (1954)
- Cumulative Sums single Observations
- Points Outside V-Mask limbs
- Put Equal Weight on Observations

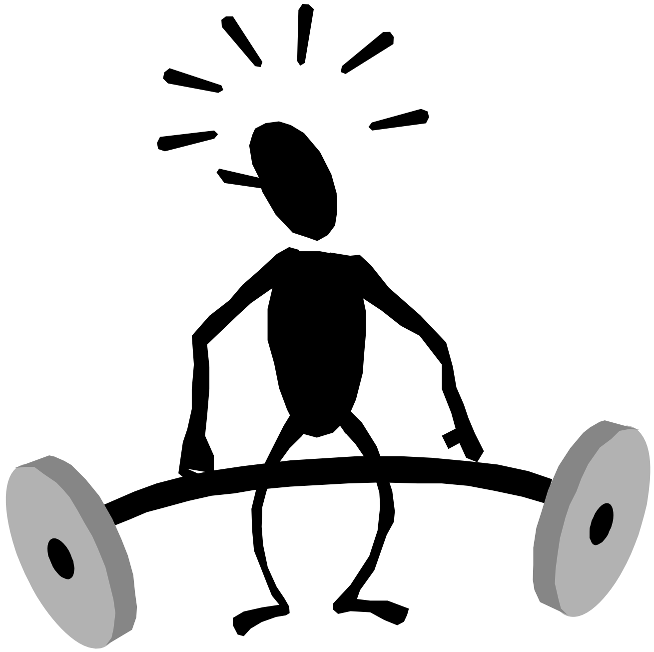
Cusum Chart Pros

- Detect “Small” Process Shifts
- Uses History



Cusum Chart Cons

- Fail to Diagnose Out-of-control Patterns
- Hard to Construct V-Mask

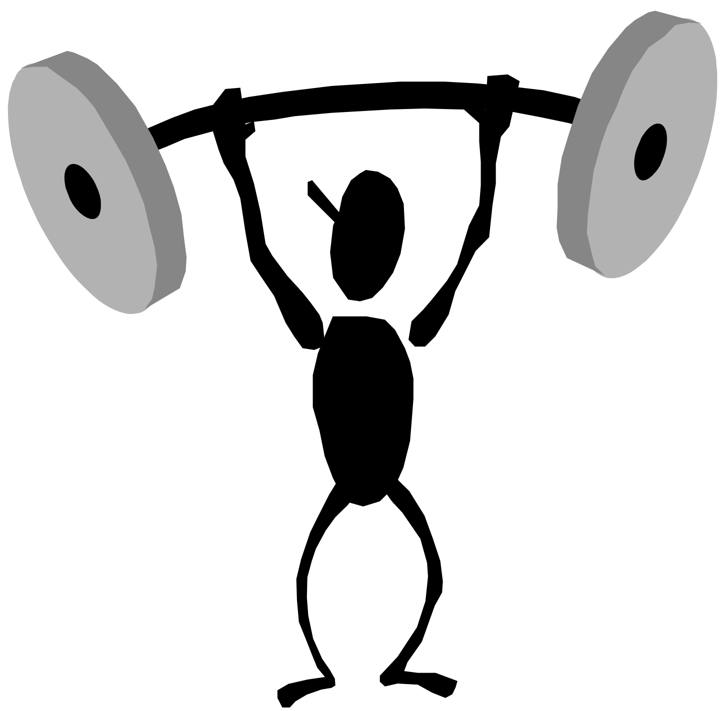


Geometric (Exponetially Weighted) Moving Average (GMA/EWMA) Control Charts

- Introduced by S.W. Roberts (1959) Time Series
- Points Fall Outside Forecast Control Limits
- Weights (w) Assign Degrees of Importance
- Resembles Shewhart & Cusum Schemes

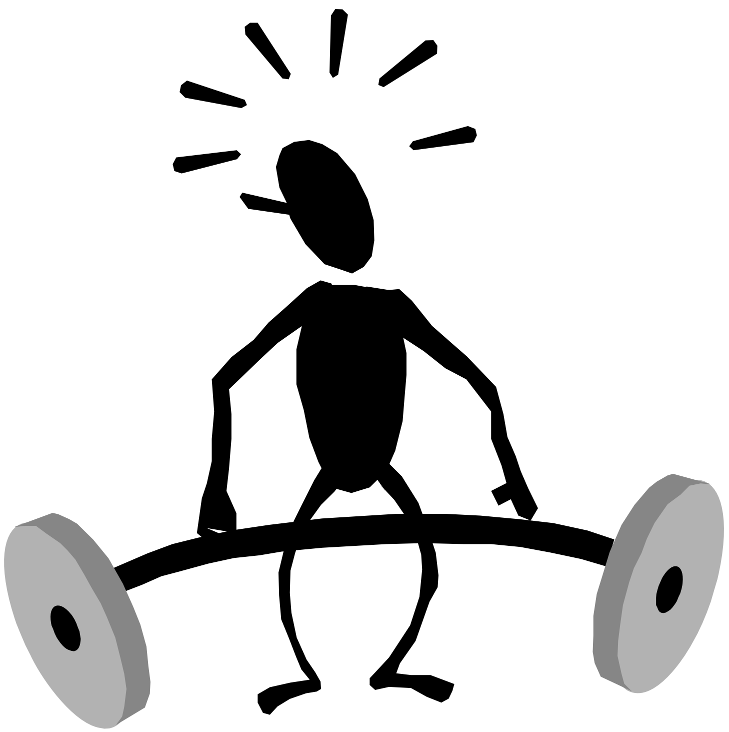
GMA(EWMA) Chart Pros

- Regular Use of History
- Approaches Shewhart if w=1,
- Approaches Cusum if w=0



GMA (EWMA) Control chart Cons

- Late in Catching Turning Points
- Inaccurate for Auto(Serial) Correlated Data
- Only One-ahead Forecasts Possible
- Hard to Monitor Changes in Weight (w)
- Allow Only One Variation Component



GMA Forecasting Model

$$F_{t+1}(w) = w * A_t + (1 - w) * F_t(w)$$

$$= F_t(w) + w * (A_t - F_t(w))$$

$$= F_t(w) + w * e_t(w)$$

where

A_t : Actual Observation at time t

$F_{t,\, t+1}(w)$: Forecast at time t, t+1 using weight factor w

$e_t(w)$: Observed error ($= A_t - F_t(w)$) at time t

The Minimum-error Weight (w') is Chosen from $\{w_1, w_2, \ldots , w_k\} \equiv \{1/T, 2/T, \ldots, k/T\}$,

Such that

$$\sum_t^T e_t^2(w') = Min\{\sum_t^T e_t^2(w_1), \sum_t^T e_t^2(w_2), \ldots, \sum_t^T e_t^2(w_k)\}$$

where $0 \leq w_1 \leq w_k \leq 1$ for k=T-1.

Sigmas (σ, σ_{gma})

$$\sigma = \sqrt{\sum_{t=1}^{T} e_t^2(w) \Big/ (T-1)}$$

T = No. of Samples

The *k*-th Lag Error Autocorrelation ($\widehat{\rho}_k$):

$$\widehat{\rho}_k = \frac{\sum_{t=k+1}^{T} [e_t(w) - \bar{e}][e_{t-k}(w) - \bar{e}]}{\sum_{t=k+1}^{T} [e_{t-k}(w) - \bar{e}]^2}$$

For k = 0, 1, 2, …, T , $\bar{e} = \frac{\sum_{t=1}^{T} e_t(w)}{T}$

If $\left|\hat{\rho}_k\right| < 2^* \sqrt{(1 + 2\sum_{t=1}^{k-1} \hat{\rho}_t^2) / T}$

Then $\hat{\rho}_k \approx 0$

Otherwise, $\hat{\rho}_k$ Estimates the Non-zero Error Autocorrelation

If the First-Lag Autocorrelation Exists (i.e., $\hat{\rho}_1 \neq 0$), Then a "Corrected" sigma (σ_c) becomes:

$$\sigma_c = \sqrt{[1 + 2 * (T - 1) * \hat{\rho}_1 / T] * \sigma^2 / T}$$

From Equation 18.3 of Box-Hunter-Hunter (1978, p. 588)

$$\sigma_{\text{gma}} = \sigma\sqrt{\frac{w}{2-w}[1-(1-w)^{2t}]}$$

$$\cong \sigma\sqrt{\frac{w}{2-w}} \text{ as } t \rightarrow \infty$$

Note: σ_c replaces σ if $\hat{\rho}_1 \neq 0$

Bayes Theorem cont.

Kalman filtering (Graves et al., 2001) and Abraham and Kartha (*ASQC Annual Technical Conf. Trans*., 1979) described ways to check weight stability.

Bayes Theorem cont.

However, control chart plots of the errors help detect weight changes (i.e., errors should exhibit a random pattern,
if not, change the weight or
the forecast method)