The Geometric Moving Average
Control Chart: A Full-Purpose
Process-Control Tool

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Focus: SPC Forecasting Control Chart
Outline

• Introduction
• Classical vs. Modern views
• Comparison between charts
• Constructing GMA charts
• Example
• Conclusions
SPC Popular Due to

- Successful in Pre-1940s Technology
- Japanese Advances in Quality
- Regain Competitive Edge in 1980s
In Catching Up We Found

- Automated Systems Expanded
- “Older Tools” Inadequate
- Modern Tools Not Fully Used
- Modern Methods Adopted/Adapted
Classical SPC “Passive”

- Describes What Happened
- Curative in Approach
- Monitors Stability
Modern SPC “Active”

• Analyzes Causes & Effect
• Predicts Future
• Gives Dynamic Feedback/Feedfront Basis for Action
Standard Control Chart Schemes

• Shewhart
• Individuals, Moving Range, & Moving Average
• CUSUM
• GMA (EWMA)
Shewhart Control Charts

- Developed by Dr. W.A. Shewhart (1924)
- Monitors Process Stability with Control Limits
- Points Outside Limits Signal Instability
- Places “Weight” on Latest Observation
Shewhart Charts Pros

- Easy to Construct
- Detects Large Process Shifts
- Helps find process change caused so that countermeasures against adverse effects may be implemented
Shewhart Chart Cons

- Ignores History
- Hard to Detect Small Process shifts
- Hard to Predict In/Out-of-Control Processes
- Really is not Process CONTROL, but is used for RESEARCH Control
Individuals, Moving Range & Moving Average Charts

- Special-Case Shewhart Charts
- Single Observations
- Control signal Like Shewhart
Individual (I) Charts

- Few Units Made
- Automated Testing on Each Unit
- Expensive Measurements
- Slow Forming Data
- Periodic (Weekly, Monthly)
- Administrative Data
Moving Range (MR) Charts

- Correspond to Shewhart Range charts
- Used with Individuals
- Plot Absolute Differences of Successive Pairs (Lags)
Moving Average (MA) Charts

• Running Average of (=4,5) Observations
• New MA Drops “Oldest” Value
• Weight Places like Shewhart (n=2 for MR; n=4,5 in MA; 0 Otherwise)
I/MA/MR Pros

- Detects Small Process Shifts
- Uses Limited History
I/MA/MR Cons

- Auto(Serial) correlated Data
  Mislead Process Stability
- Show Nonrandom Patterns with Random Data
Cusum Control Charts

- Introduced by E.S. Page (1954)
- Cumulative Sums single Observations
- Points Outside V-Mask limbs
- Put Equal Weight on Observations
Cusum Chart Pros

- Detect “Small” Process Shifts
- Uses History
Cusum Chart Cons

- Fail to Diagnose Out-of-control Patterns
- Hard to Construct V-Mask
Geometric (Exponentially Weighted) Moving Average (GMA/EWMA) Control Charts

- Introduced by S.W. Roberts (1959) Time Series
- Points Fall Outside Forecast Control Limits
- Weights \((w)\) Assign Degrees of Importance
- Resembles Shewhart & Cusum Schemes
GMA(EWMA) Chart Pros

- Regular Use of History
- Approaches Shewhart if $w=1$,
- Approaches Cusum if $w=0$
GMA (EWMA) Control chart Cons

- Late in Catching Turning Points
- Inaccurate for Auto(Serial) Correlated Data
- Only One-ahead Forecasts Possible
- Hard to Monitor Changes in Weight (w)
- Allow Only One Variation Component
GMA Forecasting Model

\[ F_{t+1}(w) = w \times A_t + (1 - w) \times F_t(w) \]

\[ = F_t(w) + w \times (A_t - F_t(w)) \]

\[ = F_t(w) + w \times e_t(w) \]

where

\( A_t \): Actual Observation at time \( t \)

\( F_{t, t+1}(w) \): Forecast at time \( t \), \( t+1 \) using weight factor \( w \)

\( e_t(w) \): Observed error \((= A_t - F_t(w))\) at time \( t \)
The Minimum-error Weight \((w')\) is Chosen from \(\{w_1, w_2, \ldots, w_k\} \equiv \{1/T, 2/T, \ldots, k/T\}\),

Such that

\[
\sum_{t}^{T} e_t^2(w') = \text{Min}\{\sum_{t}^{T} e_t^2(w_1), \sum_{t}^{T} e_t^2(w_2), \ldots, \sum_{t}^{T} e_t^2(w_k)\}
\]

where \(0 \leq w_1 \leq w_k \leq 1\) for \(k=T-1\).
Sigmas \((\sigma, \sigma_{gma})\)

\[
\sigma = \sqrt{\frac{\sum_{t=1}^{T} e_t^2 (w)}{(T - 1)}}
\]

\(T = \text{No. of Samples}\)
The $k$-th Lag Error Autocorrelation

\( \hat{\rho}_k \):

\[
\hat{\rho}_k = \frac{\sum_{t=k+1}^{T} [e_t(w) - \bar{e}] [e_{t-k}(w) - \bar{e}]}{\sum_{t=k+1}^{T} [e_{t-k}(w) - \bar{e}]^2}
\]

For $k = 0, 1, 2, \ldots, T$,

\[
\bar{e} = \frac{\sum_{t=1}^{T} e_t(w)}{T}
\]
If $|\hat{\rho}_k| < 2^* \sqrt{(1 + 2\sum_{t=1}^{k-1} \hat{\rho}_t^2)} / T$

Then $\hat{\rho}_k \approx 0$

Otherwise, $\hat{\rho}_k$ Estimates the Non-zero Error Autocorrelation
If the First-Lag Autocorrelation Exists (i.e., \( \hat{\rho}_1 \neq 0 \)), Then a “Corrected” sigma (\( \sigma_c \)) becomes:

\[
\sigma_c = \sqrt{[1 + 2 \times (T - 1) \times \hat{\rho}_1 / T] \times \sigma^2 / T}
\]

From Equation 18.3 of Box-Hunter-Hunter (1978, p. 588)
\[ \sigma_{gma} = \sigma \sqrt{\frac{w}{2-w}} [1 - (1-w)^{2t}] \]

\[ \cong \sigma \sqrt{\frac{w}{2-w}} \text{ as } t \to \infty \]

Note: \( \sigma_c \) replaces \( \sigma \) if \( \hat{\rho}_1 \neq 0 \)
Bayes Theorem cont.

Kalman filtering (Graves et al., 2001) and Abraham and Kartha (*ASQC Annual Technical Conf. Trans.*, 1979) described ways to check weight stability.
Bayes Theorem cont.

However, control chart plots of the errors help detect weight changes (i.e., errors should exhibit a random pattern, if not, change the weight or the forecast method)